How Do We Understand the Coriolis Force?

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ABSTRACT

The Coriolis force, named after French mathematician Gaspard Gustave de Coriolis (1792–1843), has traditionally been derived as a matter of coordinate transformation by an essentially kinematic technique. This has had the consequence that its physical significance for processes in the atmosphere, as well for simple mechanical systems, has not been fully comprehended. A study of Coriolis's own scientific career and achievements shows how the discovery of the Coriolis force was linked, not to any earth sciences, but to early nineteenth century mechanics and industrial developments. His own approach, which followed from a general discussion of the energetics of a rotating mechanical system, provides an alternative and more physical way to look at and understand, for example, its property as a complementary centrifugal force. It also helps to clarify the relation between angular momentum and rotational kinetic energy and how an inertial force can have a significant affect on the movement of a body and still without doing any work. Applying Coriolis's principles elucidates cause and effect aspects of the dynamics and energetics of the atmosphere, the geostrophic adjustment process, the circulation around jet streams, the meridional extent of the Hadley cell, the strength and location of the subtropical jet stream, and the phenomenon of "downstream development" in the zonal westerlies.

"To understand a science it is necessary to know its history." (August Comte, 1798–1857)

1. Introduction

Although the meteorological science enjoys a relative wealth of colorful figures and events, from past and present, the educators rarely make use of this historical dimension, as was recently pointed out in this journal by Knox and Croft (1997). A case where a historical approach proves to be illuminating is in the teaching of the Coriolis force, named after French mathematician Gaspard Gustave Coriolis (1792–1843). On a rotating earth the Coriolis force acts to change the direction of a moving body to the right in the Northern Hemisphere and to the left in the Southern Hemisphere. This deflection is not only instrumental in the large-scale atmospheric circulation, the development of storms, and the sea-breeze circulation (Atkinson 1981, 150, 164; Simpson 1985; Neumann 1984), it can even affect the outcome of baseball tournaments: a ball thrown horizontally 100 m in 4 s in the United States will, due to the Coriolis force, deviate 1.5 cm to the right.

2. The mathematical derivation

The "Coriolis acceleration," as it is called when m is omitted, is traditionally derived by a coordinate transformation. The relation between the acceleration of a vector, **B**, in a coordinate system fixed relative to the stars (f) and a system, (r), rotating with an angular velocity, ω, is

$$\left(\frac{d\mathbf{B}}{dt}\right)_f = \left(\frac{d\mathbf{B}}{dt}\right)_r + \omega \times \mathbf{B}. \qquad (1)$$

The procedure is to apply (1) first to the position vector r, then to its velocity v to get the relative ve-

locity $\mathbf{v}_r$, combine the expressions, and arrive at an expression for the absolute acceleration a,

$$\mathbf{a} = \mathbf{a}_r + 2\boldsymbol{\omega} \times \mathbf{v}_r + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times r), \qquad (2)$$

which for a rotating observer is composed of the observed acceleration $\mathbf{a}_r$; the Coriolis acceleration $2\boldsymbol{\omega} \times \mathbf{v}_r$, which only depends on the velocity; and the centrifugal acceleration $\boldsymbol{\omega} \times (\boldsymbol{\omega} \times r)$, which only depends on the position (French 1971, 522; Pedlosky 1979, 17–20; Gill 1982, 73). For a given horizontal motion the strongest horizontal deflection is at the poles and there is no horizontal deflection at the equator; for vertical motion the opposite is true (Fig. 1).

3. Frustration and confusion?

The mathematics involved in the derivation of the Coriolis force is quite straightforward, at least in comparison with other parts of meteorology, and cannot explain the widespread confusion that obviously surrounds it [for a recent example see an article by Kearns (1998) in *Weatherwise*]. The late Henry Stommel appreciated the sense of frustration that overcomes students in meteorology and oceanography who encounter the "mysterious" Coriolis force as a result of a series of "formal manipulations":

> Clutching the teacher's hand, they are carefully guided across a narrow gangplank over the yawning gap between the resting frame and the uniformly rotating frame. Fearful of looking down into the cold black water between the dock and the ship, many are glad, once safely aboard, to accept the idea of a Coriolis force, more or less with blind faith, confident that it has been derived rigorously. And some people prefer never to look over the side again. (Stommel and Moore 1989).

This article will suggest that the main problem with the teaching of the Coriolis force does not lie so
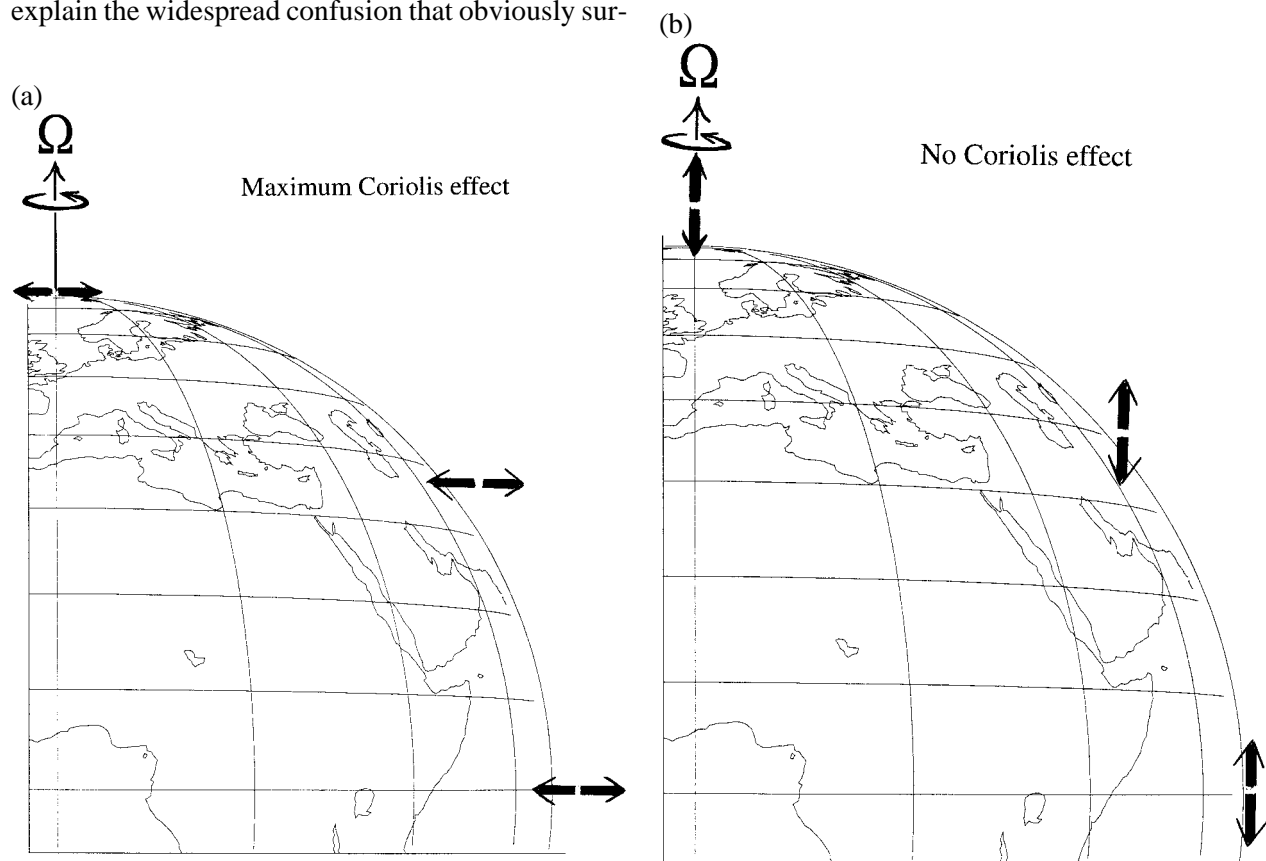
(a)

(b)

FIG. 1. Since the Coriolis force is the cross product between the rotational vector of the earth (Ω) and the velocity vector, it will take its maximal values for motions perpendicular to the earth's axis (a) and vanish for motions parallel to the earth's axis (b). Affected by a maximal Coriolis force is, for example, air rising equatorward (or sinking poleward) in the midlatitudes. The Coriolis force vanishes for air rising poleward (or sinking equatorward). Horizontal west–east winds on the equator are indeed affected by the Coriolis force, although the deflection is completely in the vertical direction.

much with the mathematics of the derivation, but the purely kinematic nature of the derivation. It readily provides the "approved" answer, but the price to pay is a pedagogical difficulty to bridge the gap between this formalistic approach and a genuine physical understanding.

4. The mechanical interpretation of the Coriolis force

Many textbooks are anxious to tell the student that the Coriolis force is a "fictitious force," "an apparent force," "a pseudoforce," or "mental construct." The centrifugal force, however, although equally fictitious, is almost always talked about as a force. This leaves the impression that some fictitious forces are more fictitious than others.

Some textbooks, with an ambition to avoid "formal manipulations" and appeal to the student's intuition, derive the Coriolis force (in two dimensions) for a moving body on a turntable, separately for tangential and radial movements, making use of the variations of the centrifugal force, ωr^2, and conservation of angular momentum, $\omega^2 r$, respectively. However, problems arise when the physical conditions under which this motion takes place are not taken into account or addressed. In most rotating systems the dominating force is not the Coriolis force but the centrifugal

force. A man walking at a pace of 1 m s^{-1} at a distance of 3 m from the center of rotation on a turntable, making one revolution in 2 s, will experience a centrifugal acceleration that is five times stronger than the Coriolis acceleration. Only at a distance of 0.6 m from the center are the two forces of equal strength. The centrifugal effect is eliminated if there is no interaction between the rotating disk and the body, like a ball rolling on the disk without friction. But then the problem is brought back to the traditional kinematic approach with transformations between coordinate systems.

Conservation of angular momentum only applies when there is no torque about the vertical axis. A person moving on the rotating surface will exert a torque on it through friction and the conservation will therefore only apply to the man and the turntable together, and angular momentum will not be conserved for either of them. For quite natural reasons these problems vanish if we consider movements on the rotating earth.

Due to its ellipsoid shape, the gravitational forces balance the centrifugal force on any body, as long as it does not move (Fig. 2). When it moves, the balance is altered. For an eastward movement the centrifugal force is increased and the body is deflected toward the equator, to the right of the movement. For a westward movement the centrifugal force is weakened and can no longer balance the gravitational force, which is the physical force that moves the body in the poleward
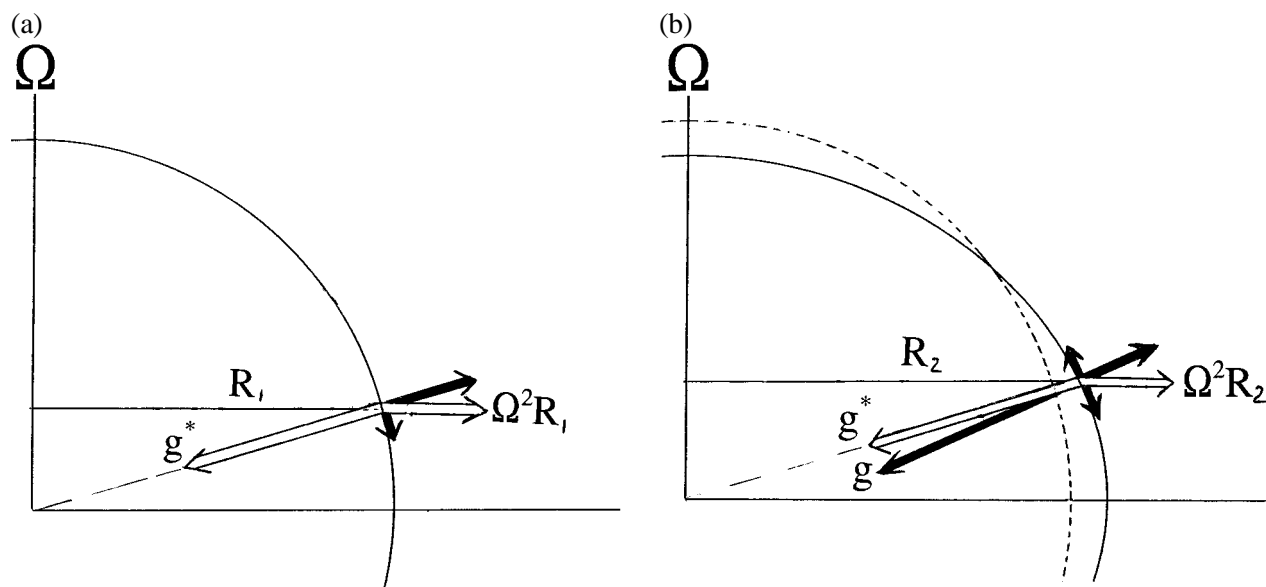
(a)

(b)

Fig. 2. The balance between the centrifugal force and the gravitation g*. In the early stages of the earth's development (a) the centrifugal force pushed the soft matter toward the equator. A balance was reached when the equatorially directed centrifugal force balanced the poleward gravitational force and changed the spheroidal earth into an ellipsoid (b).

direction to the right of the movement (Durran 1993; Durran and Domonkos 1996).

The same result follows from a consideration of conservation of angular momentum of a body moving on the earth without being affected by torques. For poleward (equatorward) movements a decrease (increase) of the distance to the earth's axis of rotation is associated with an increase of the westerly (easterly) zonal speed. These deflections (pointing to the left of the motion on the Southern Hemisphere) are manifestations of the Coriolis force. The application of angular momentum conservation has, however, given rise to some misunderstandings.

Some textbooks (and educational sites on the World Wide Web) explain qualitatively the Coriolis deflection of a meridional movement as a consequence of the air's origin at another latitude where its velocity due to the earth's rotation was different (e.g., Battan 1984, 117–118). But this does not relate to the principle of conservation of angular momentum, but to conservation of absolute velocity. This misunderstanding is deceptive because it yields a deflection in the right direction, but only explains half of the Coriolis acceleration $\boldsymbol{\omega} \times \mathbf{v}_r$ instead of $2\,\boldsymbol{\omega} \times \mathbf{v}_r$. The seriousness of the mistake lies not primarily in the numerical error, but in the confusion between two fundamental mechanical principles: conservation of linear momentum and conservation of angular momentum. This potential misunderstanding is acknowledged by Eliassen and Pedersen (1977, 98), who make it clear how two kinematic effects each contribute half of the Coriolis acceleration: relative velocity and the turning of the frame of reference. This can also be understood from simple kinematic considerations (Fig. 3).

5. Foucault, Ferrel, and Buys Ballot

It was only in the late eighteenth century that it was realized that angular momentum conservation, which so far had only been applied to celestial mechanics (Kepler's second law), could also be applied to mechanics. So when George Hadley in 1735 realized that the earth's rotation deflects air currents, he discussed it only in terms of conservation of absolute velocity (Lorenz 1967, 2, 61; Lorenz 1969, 5; see facsimile of Hadley's paper in Shaw 1979). This needs to be emphasized since Gill in his book on ocean and atmospheric dynamics repeatedly claims that Hadley indeed made use of conservation of angular momentum (Gill 1982, 23, 189, 369, 506, 549). Although Gill quotes

Hadley's mathematics in detail (on p. 23) he does not seem to realize that it relates to conservation of absolute velocity. Similar misinterpretations of Hadley's work can be found in other authoritative texts, most recently in this journal (Lewis 1998, 39, 53).

When the effect of the earth's rotation was debated by scientists during the eighteenth century, it was always in Hadley's terms of absolute velocity. There was no notion of any Coriolis force until the mid-nineteenth century when Jean Bernard Léon Foucault demonstrated that a simple pendulum would be deflected by the earth's rotation and at latitude φ make a full revolution in one pendulum day, which is $24\ \text{h}/\sin\varphi$, double the time for an inertia oscillation (Fig. 4). This discovery attracted wide scientific and popular attention and, together with reading Newton's and Laplace's works, inspired William Ferrel in 1856 to conclude that the direction of the wind is parallel to the isobars, its strength dependent on the latitude and the horizontal pressure gradient (Khrgian 1970, 222; Kutzbach 1979, 36–38). Independently of Ferrel, the Dutch meteorologist C. H. D. Buys Ballot in 1857 published his rule based on empirical data according to which low pressure is to the left if you have the wind at your back (Snelders and Schurrmans 1980).

Foucault was not an academic and there are good reasons to assume that he had never heard about Co-
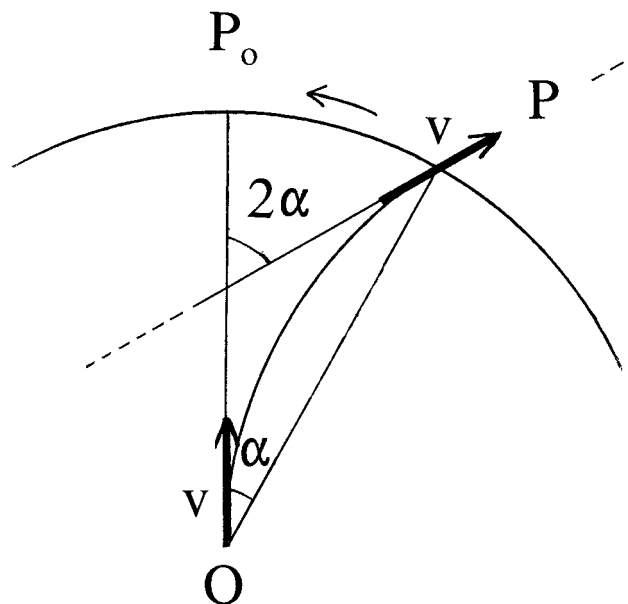

Fig. 3. A straight movement from O to P on a rotating disc/turntable/merry-go-round follows a curved path. The track is along the curved arc and the direction of the motion (relative to the fixed axis) has turned through an angle 2 α while the system has rotated only an angle, α. (From an idea by R. S. Scorer.)

riolis when he designed his experiment. So where does Coriolis enter? In a debate in this journal 30 years ago (Burstyn 1966; Landsberg 1966; Haurwitz 1966; Jordan 1966) the question was indeed raised if he has any place in the meteorological science? One participant, who was familiar with Coriolis's original paper through Dugas (1955, 374–383), noted that he derived his force "not by transforming coordinates or conservation of angular momentum, either of which might suggest themselves to us, but by considering the energetics of the system, which appears to us to be doing it the hard way" (Burstyn 1966). This is the closest any meteorologist appears to have come to find out what Coriolis actually did. Fortunately his papers have recently been reprinted (Coriolis 1990). They reveal that his work is not only of historic interest, but also opens up new ways to look at the deflective force and its role in atmospheric dynamics.

6. Gaspar Gustave Coriolis

Gaspard Gustave Coriolis was born on 21 May 1792 in Paris to a small aristocratic family that was impoverished by the French Revolution (Fig. 5). The young Gaspard early showed remarkable mathematical talents. At 16 he was admitted to the École Polytechnique where he later became a teacher. As one of his biographers noted, it was this teaching that inspired his work (Costabel 1961; see also Tourneur 1961; Société Amicale des Anciens Élèves de l'Ecole Polytechnique 1994, 7, 122; Lapparent 1895). The education of mechanics in France at the time was dominated by statics, which was suited only for problems related to constructional work, not for machines driven by water or wind. Lagrange's mechanical theories were criticized, not because they were wrong, but because they were difficult to apply to practical problems. A movement developed with the chief goal to raise the education of workers, craftsmen, and engineers in "méchanique rationelle." Coriolis was among the first to promote the reform and in 1829 he published a textbook, *Calcul de l'Effet des Machines (Calculation of the Effect of Machines)*, which presented mechanics in a way that could be used by industry. It was only now that the correct expression for kinetic energy, $m v^2/2$, and its relation to mechanical work became established (Grattan-Guinness 1997, 330, 449; Kuhn 1977).
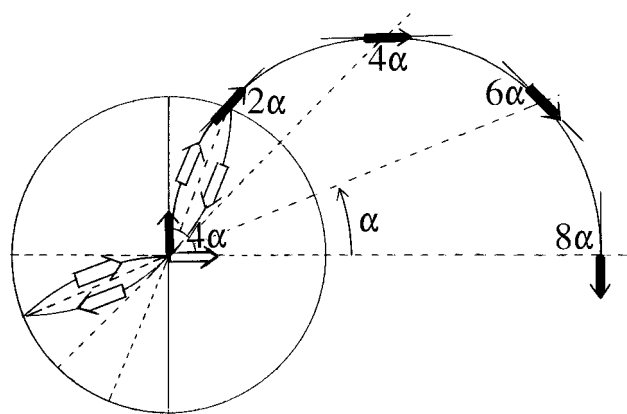

FIG. 4. A graphical illustration, an extension of Scorer's idea in Fig. 3, of why it takes half the time for a body to make an inertial oscillation (12 h/sinφ where φ is the latitude) than for the arch of a Foucault pendulum to make a full rotation 24 h/sinφ). One body is moving under inertia (full arrows), the other is a pendulum bob (open arrows), both seen from above and following the earth's rotation of α/τ (degrees/time interval). During a time interval 4 τ, under which the body following an inertia circle has changed its direction by 8 α, the pendulum bob has only turned half of that or 4 α. The heart of the matter is that the effect of the Coriolis deflection is reversed when the swinging bob moves in the opposite direction, while this is not the case for the inertia movement. The explanation also makes it clear that it is irrelevant if the bob is moving as a pendulum or in any other way, as long as the direction is constant in space and is reversed periodically. (The change of direction per swing in the figure is grossly exaggerated.)
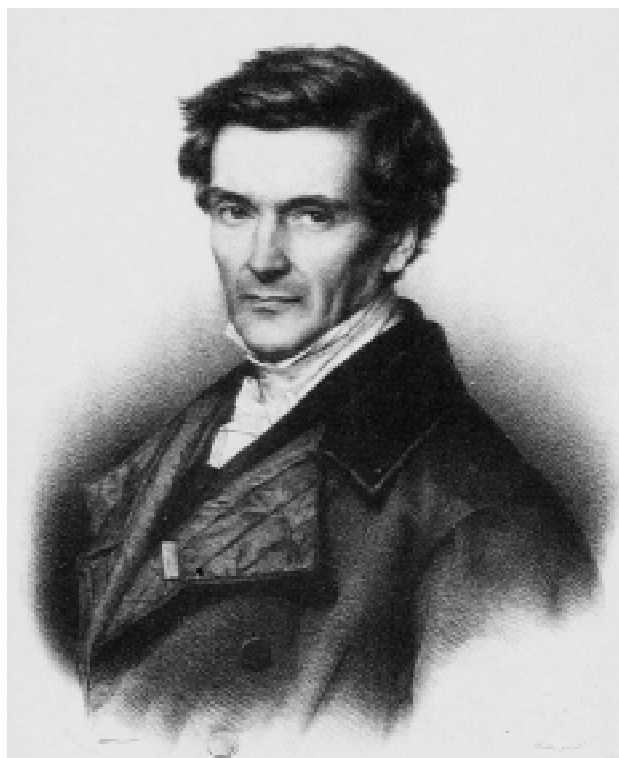

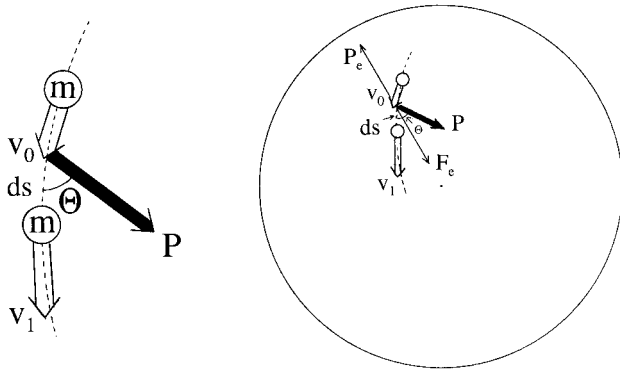FIG. 5. Gaspard Gustave Coriolis (1792–1843).

FIG. 6. Coriolis's first theorem: a body, m, on a rotating turn-table moving with a speed, $\mathbf{v}_0$, is subject to a force, P, and displaced along a trajectory, ds, and accelerates to $\mathbf{v}_1$. The change in kinetic energy, corresponding to the change of potential energy, is due to the work done by the driving force P along the projection of the distance ds where Θ is the angle between P and ds. (To make the dynamic discussion complete, Coriolis also considered the centripetal force P_e and the balancing centrifugal force F_e, both acting to keep the body in a fixed position in the absence of a driving force P. Both F_e and P_e cancel out in the energy equation.)

During the following years Coriolis worked to extend the notion of kinetic energy and work to rotating systems. The first of his papers, "Sur le principe des forces vives dans les mouvements relatifs des machines" ("On the principle of kinetic energy in the relative motion in machines"), was read to the Académie des Sciences (Coriolis 1832). Three years later came the paper that would make his name famous, "Sur les équations du mouvement relatif des systèmes de corps" ("On the equations of relative motion of a system of bodies"; Coriolis 1835).

Coriolis's papers do not deal with the atmosphere or even the rotation of the earth, but with the transfer of energy in rotating systems like waterwheels. The 1832 paper established that the relation between potential and kinetic energy for a body, m, moving with a velocity, $\mathbf{v}_0$, affected by a force, P, which makes it accelerate to a velocity, $\mathbf{v}_1$, is the same in a rotating system as in a nonrotational (Fig. 6):

$$m\mathbf{v}_1^2/2 - m\mathbf{v}_0^2/2 = \int P \cos \Theta \, ds, \qquad (3)$$

where Θ is the angle between P and ds. Coriolis applied this relation on problems in nineteenth-century technology; we can, for example, relate it to the increase of speed and kinetic energy of a satellite falling toward the earth. The work is done by the gravitational force along the projection on the satellite's trajectory.

Three years later, in 1835, Coriolis went back to analyze the relative motion associated with the system, in particular the centrifugal force. It is directed perpendicular to the moving body's trajectory (seen from a fixed frame of reference), which for a stationary body is radially out from the center of rotation. For a moving body this is not the case; it will point off from the center of rotation. The centrifugal force can therefore be decomposed into one radial centrifugal force, $m\,\boldsymbol{\omega}^2\,\mathbf{r}$, and another, $-2\,m\,\boldsymbol{\omega}\,\mathbf{v}$, the "Coriolis force." It is worth noting that Coriolis called the two components "forces centrifuges composées" and was interested in "his" force only in combination with the radial centrifugal force to be able to compute the total centrifugal force.
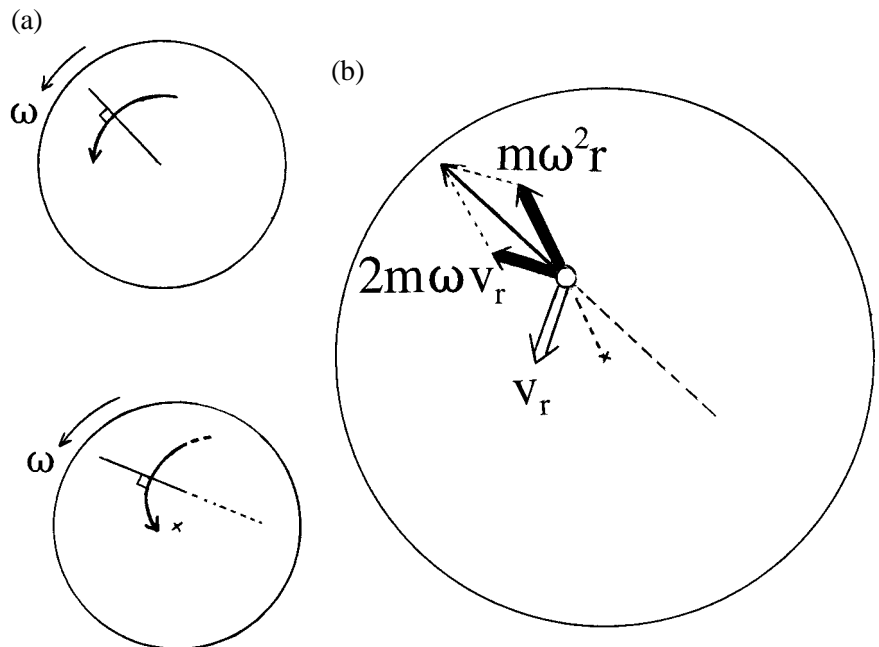

FIG. 7. Coriolis's second theorem is most easily understood when the rotating system is viewed first from a fixed frame of reference (a), then in the rotating frame of reference (b). The total centrifugal force acting on the body m moving with a velocity $\mathbf{V}$ is directed perpendicular to the tangent of the movement, along the radius of curvature (Fig. 6a). It can be decomposed into two centrifugal forces; one $m\,\boldsymbol{\omega}^2\,r$, directed from the center of rotation and a second, $-2m\mathbf{V}, \omega$, the Coriolis force, perpendicular to the relative motion $\mathbf{V}_r$ (Fig. 6b).

It is significant that Coriolis did not make use of angular momentum conservation. His two theorems actually help us to understand that this important principle has its explanatory limitations.

7. Kinetic energy and angular momentum

It is a common misconception that kinetic energy and angular momentum, if not the same, at least vary synchronously, just like linear momentum and kinetic energy. To show that this is not the case, let us make use of one of the popular actors on the pedagogical scene: the rotating ice skater. She increases her rotation when she horizontally moves her arms inward. With no friction against ice and air she conserves her angular momentum. This is the standard argument. More rarely addressed is the question of what happens to her rotational kinetic energy. One might be tempted to assume that it also remains conserved, but this is wrong. It will increase significantly.

A short and elegant proof is found in one of R. Feynman's lectures on physics: with her arms stretched out the ice skater has a moment of inertia, I; an angular velocity, ω; and an angular momentum, $L = I\omega$, which remains constant. The rotational kinetic energy, $K = I\omega^2/2$ or $L\omega/2$, increases with increasing rotation, when L remains unchanged (Feynman et al. 1977, 19-8).

But from where does the extra kinetic energy come? Contracting one's arms while standing still does not constitute any work in a mechanical sense; while rotating one has to apply a force inward to counter the centrifugal force; the extra energy comes from the work done along this force. This follows exactly from Coriolis's first theorem: the increase in rotation is achieved by the work done by her muscles when she pulled in her arms.

It is not correct to say that the ice skater "makes use of" the Coriolis force "in order to" increase her rotation. Since the Coriolis force is always directed perpendicular to the movement of a body, it can only change its direction, not its speed and kinetic energy: it does no work. Nor is it quite true that she increases her rotation "in order to" conserve angular momentum. Even if friction from the ice and the air will slightly decrease her angular momentum, she will still increase her rotation and kinetic energy. A satellite entering the earth's outer atmosphere will decrease its angular momentum, due to friction, but, as mentioned earlier, still increase its speed and kinetic energy. Contrary to what might intuitively be expected, the increase will be proportional to the strength of the resisting frictional force (French 1965, 471–473).

The limitations of trying to interpret all kinds of rotating motion solely from an angular momentum perspectives become clear if we perform another experiment with the (by now exhausted) ice skater. Let her carry heavy weights in her hands. When she contracts her arms, she will still conserve angular momentum, but this will not prepare us for what happens next: at some stage in the contraction she will be unable to continue to move her arms inward. This occurs when the outward centrifugal force has increased so much that it balances the centripetal force from her arm muscles.

An interesting situation arises when she slackens her arms. The centrifugal force drives them outward and her rotational kinetic energy is converted into potential energy. Work is done, not by the centrifugal force, but by her muscles, doing negative work. "Negative work" is a well-established concept in mechanics and simply means the work done when a force moves in a direction opposite to that in which it works. When a ball passively rolls uphill, inertia might "do the job," but the conversion from kinetic to potential energy is done by gravity, doing negative work. So far the author has seen the notion of negative work explicitly mentioned only once in the meteorological literature (Ertel 1938, 48).

Coriolis's first and second theorems put the deflective force into a dynamical context and make clear what it does—and does not do. All this is possible because Coriolis avoided a kinematic approach and instead derived "his" force through a dynamic analysis of a mechanical system. This is a circumstance that is more remarkable than it appears at first glance.

8. The development of kinematics

In 1834, at the same time as Coriolis worked on his derivations, one of his colleagues at the École Polytechnique, André-Marie Ampère, published a major philosophical treatise, "Essai sur la Philosophie des Sciences." He noted that from the experience by earlier scientists, from Kepler to Euler, it was found to be possible to study motions without necessarily considering the forces that create or result from this motion. Ampère suggested that this approach could form a new branch of mechanics, which he called cinématique. Kinematics was soon accepted as a new

discipline. It was soon developed to higher levels, among others by George Stokes, working on his equations for fluids and solids (Smith and Wise 1989, 199, 360–372; Koetiser 1994). From fluid mechanics the step was not far to dynamic meteorology.

The use of kinematics in dynamic and synoptic meteorology includes not only parameters like divergence and vorticity (absolute, relative, and potential), but also more complicated relations like the geostrophic and thermal wind relations, the omega equation, Sutcliffe's equation, the Q vector, etc. Many of them are based on constraints like balance conditions or conservation properties and are useful to analyze and predict atmospheric movements: "When the causative forces are disregarded, motion descriptions are possible only for points having constrained motion, that is, moving on determined paths. In unconstrained or free motion the forces determine the shape of the path" (*Encyclopaedia Britannica*, s.v. "kinematics"). Although the use of constraints makes kinematics potentially less powerful a tool than dynamics (Hess 1979, 198), there is no contradiction between the two. They complement each other: the former describes "how," the latter "why."

As Knox and Croft (1997, 903) point out, thinking in terms of divergence, vorticity, development equations, etc., ties well with "the teddy bear of the good ol' equations of motion," in particular when the student gets confused with the former. One source of such confusion is misinterpretations of kinematic models, as pointed out by Holton (1993) and Persson (1996, 1997). A common malpractice is to defy the limitations of kinematics and try to deduce causal relations. This leads easily to metaphysical reasonings like, "In order to maintain geostrophic balance, the wind has to . . .". Since in a kinematic system any variable can be appointed as a "cause," it is not more true to say that the divergence term in the vorticity equation is the cause of changes in the vorticity term, than it is to say that a change of vorticity is the cause of divergence in the wind field. Further confusion comes from the introduction of kinematical and statistical concepts from turbulence theory, for example, "Reynolds stresses," to represent physical mechanism, in spite of warnings from Pedlosky (1979, 172) among others.

It goes beyond the scope of this article to discuss the misuse of kinematic conceptual models, except when the Coriolis force is directly involved. This is the case with the pedagogical use of angular momentum conservation to explain certain features in the general circulation.

9. Angular momentum in the atmosphere

The principle of angular momentum conservation is dependent on the condition that there is no net torque in the direction of the rotation, like friction or pressure gradient forces. The conservation therefore applies only to the atmosphere as a whole, since the global rate of working of the pressure gradient force is zero. It does not apply to parts of it where local pressure gradients exert torques. This was well known by the meteorological generation before the Second World War. As a consequence, they were critical to angular momentum conservation as a model for the large-scale circulation. Exner (1917) and Haurwitz (1941) published tables to show how a parcel moving meridionally just 10° of latitude under conservation of angular momentum would experience a wind increase of 100 m s⁻¹ or more (Table 1).

The leading British meteorologist of the time, David Brunt, regarded the whole idea of air moving from one latitude to another while retaining its original angular momentum, as "highly misleading" since air

TABLE 1. Increase in speed of a ring of air displaced 10° meridionally under conservation of angular momentum (after Exner 1917, 24, 179–81; Haurwitz 1941, 121). Both Exner and Haurwitz concluded that large-scale meridional displacements of air masses under conservation of angular momentum hardly occurred in the atmosphere, at least not at higher latitudes.

From lat	To lat	Velocity u m s^{-1}	To lat	Velocity u m s^{-1}
90°	80°	−81.5		
80°	−70°	−117.3	90°	∞
70°	60°	−123.2	80°	234.7
60°	50°	−118.3	70°	179.8
50°	40°	−105.3	60°	151.1
40°	30°	−87.7	50°	125.9
30°	20°	−65.9	40°	98.9
20°	10°	−41.0	30°	71.5
10°	0°	−14.0	20°	43.0
0°			10°	14.2

set in motion cannot travel to another latitude unless it is guided by "a suitable arranged" pressure gradient. If there were no pressure gradients, the air would be constantly deviated due to the Coriolis force, describe an inertia oscillation, and return to its latitude. Air moving meridionally 20 m s^{-1} at 60° would turn back after 160 km (Brunt 1941, 404–405). The reason a hemispheric Hadley circulation is not possible is not that it would be baroclinicly unstable in the midlatitudes, as is often stated, but that air conserving angular momentum will be unable to move very far poleward. As Durran (1993) recently pointed out, an inertial oscillation is also a "constant angular momentum oscillation."

Carl-Gustaf Rossby, although at first a proponent of explaining the general circulation in terms of angular momentum conservation (Rossby 1941), soon became skeptical when he realized that the existence of zonal pressure gradients violated the principle. Those who tried to avoid the problems by considering longitudinal averages of pressure and wind in a hemispheric ring of air did not cut any ice with him. They were compared with Ptolemaic astronomers who added epicycles to their system to explain the motions of the planets (Rossby 1949b, 18, 23, 26). This was the start of the famous Rossby–Starr–Palmén debate, which soon came to focus on other aspects of the general circulation (Lewis 1998).

It remains a topic for historical research to find out why and how angular momentum conservation, in spite of the previous scientific objections, in the late 1940s became established as the key to understand the general circulation. Although its basis was never theoretically challenged, except by James (1953), its has been accepted with mixed feelings. Whereas Hartmann (1994, 150) regards angular momentum conservation as a "heavy constraint" on the atmospheric movements, others see it as "a clearly hopeless description" (James 1994, 81–82) due to the unrealistically strong zonal winds that would develop at middle and high latitudes, a point made already by Helmholtz in the 1880s (Lorenz 1967, 66–71).

10. Coriolis's approach applied to the atmosphere

So far we have seen that the dynamics of an ice skater can be understood to only a limited extent exclusively from an angular momentum point of view, while Coriolis's two theorems cover alternative and complementary aspects. This will prove valuable for the understanding of atmospheric processes, in particular for the wind and pressure relation.

a. Geostrophic adjustment

Lorenz (1967, 29, 65) has commented on a frequent tendency among meteorologists to assume that the wind field is somehow produced by the height field in a simple one-way process and to overlook that they determine one another through mutual effects. The common textbook discussion relates to the acceleration of a (subgeostrophic) wind into a confluent isobaric field. Rarely discussed is what happens in a diffluent isobaric field: the pressure gradient force weakens and the (supergeostrophic) wind is deflected toward higher pressure by the Coriolis force. The mutual wind and pressure adjustment is a consequence of the mass and energy transport across the isobars (Uccellini and Johnson 1979; Uccellini 1990, 125).

b. Jet stream dynamics

The three-dimensional flow around a jet stream, traditionally described in kinematic terms of vorticity advection, can also be understood in Coriolis's terms of forces, energy, and in particular inertia. The mass and kinetic energy released at the entrance of a jet stream will be rapidly carried through the jet by the flow itself. At the exit the velocity is reduced and the kinetic energy is converted back to the "reservoir of potential energy" (Kung 1977). The pressure pattern is slowly driven downstream while the air is rapidly passing through. Further energy discussions explain the tendency for upward motion and cyclogenesis at the right entrance and the left exit (Fig. 8). The tradi-
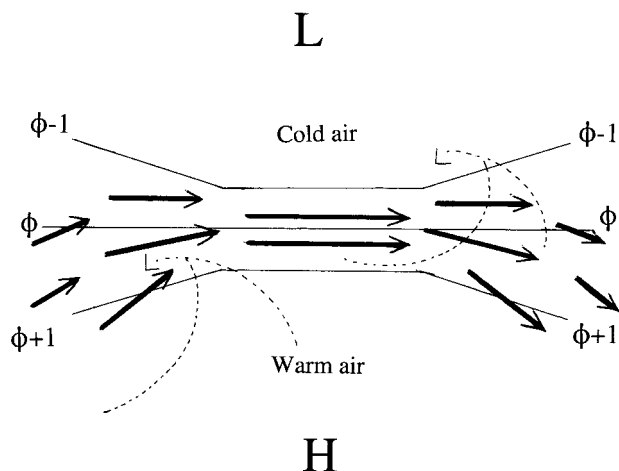

FIG. 8. A schematic illustration of a jet stream with geopotential lines, wind vectors, and two frontal systems and the inferences that can be made from it (see appendix).

tional kinematic vorticity explanation can neither account for the vertical (indirect) circulation at the exit as a response to the kinetic energy arriving under inertia from the entrance, nor can it explain the speed by which a development at the right entrance of the jet rapidly may affect and even initiate a development at the left exit. The mechanism by which the available potential energy is rapidly transported downstream, temporarily converted into kinetic energy, and then at the jetstream exit converted back to available potential energy is what forecasters since long ago have recognized as one of the links in a "downstream development" chain, which can spread over half of the hemisphere in less than a week.

c. Energy balance

The process of conversion between potential and kinetic energy in the free atmosphere is adiabatic and reversible (van Mieghem 1973). There are widespread misconceptions that it is irreversible (see, e.g., Rossby 1949a, 163; Carlson 1991, 16, 109, 114, 443; Lewis 1998) and can only go back to potential energy after having been dissipated by friction to heat (Petterssen and Smedbye 1971; Carlson 1991, 122). The well-known energy boxes (Fig. 9) only show the net conversion between two almost equally strong fluxes of potential and kinetic energy, and the long-term ultimate and irreversible dissipation of kinetic energy into heat (Lorenz 1967, 103; van Mieghem 1973, 157,

FIG. 9. Two ways to depict the "energy boxes" of the atmosphere's general circulation: (a) the conventional, which shows the net fluxes, but might easily be misunderstood to mean that the flux can only go one way; (b) a pedagogically more secure representation where it clearly comes out that the net flux is the difference between two almost equally large fluxes in both directions.

Uccellini 1990, 125; Kung 1971, 61; Peixoto and Oort 1992, 311). Turbulent and frictional dissipations are important sinks for kinetic energy in the boundary layer but not in the free atmosphere where most of the kinetic energy is converted back to potential energy. Work is done, but not by the Coriolis force, as stated, for example, by Starr (1969, 198, 256), but by the pressure gradient force, doing negative work. The fact that large-scale motion can convert kinetic energy into potential energy distinguishes it from three-dimensional turbulence, where the total kinetic energy is constant but redistributed among the scales. When L. F. Richardson wrote his famous verse about "Big whirls have little whirls/that feed upon their velocity/And little whirls have lesser whirls/and so on to viscosity," he thought about normal turbulence, not the large-scale circulation as is implied by some textbooks (e. g., Wallace and Hobbs 1977, 437).

d. The subtropical jet and the Hadley cell

One of the strongest and most persistent wind systems is the subtropical jet on the poleward side of a strong Hadley cell (Fig. 10). Fifty years after its discovery it is still, according to a recent article in *Weatherwise*, "the most frequently slighted" of all the jet streams and is in need of "a new publicity agent" (Grenci 1997). Curiously, there does not seem to be any consensus among the experts about its mechanism (Wiin-Nielsen and Chen 1993, 151) or why it is not stronger (Hartmann 1994, 153). This might be due to attempts to explain it along the line of angular momentum conservation and transport. Coriolis's approach would lead us to seek the explanation in the north–south pressure gradient in the Hadley circulation, which drives the air northward and increases its speed, while at the same time the Coriolis force deflects this increased wind toward the east. It is the pressure gradient force that increases the wind and kinetic energy, not the Coriolis force, as stated, for example, by Starr (1954, 271) and Wiin-Nielsen and Chen (1993, 161). A rather weak pressure gradient of 5 hPa over 10 latitude degrees yields an acceleration of 0.4 mm sec^{-2}, which does not appear much, but after one day has resulted in a wind speed of 40 m s^{-1} and transported air 2000 km, a distance equal from the equator to 20° latitude.

When the supergeostrophic wind on the poleward side of the Hadley cell adjusts geostrophically, zonal pressure gradients are set up or enhanced that prevent angular momentum from being conserved. With the earth's particular differential heating and rotation, this
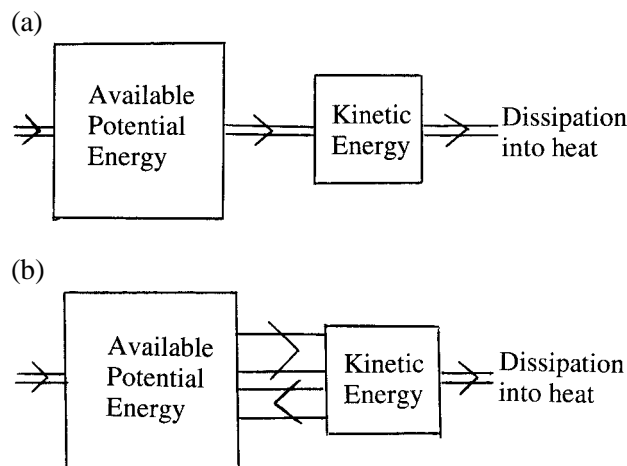
deflection occurs at about 30° latitude and this determines its meridional extent of the Hadley cell. On another planet, with weak differential heating and/or faster rotation, the Hadley cell will be confined to the equatorial band where there will be a strong equatorial jet stream, as shown in illuminating experiments by Williams (1988). A simple mechanic model that illustrates this is the previously discussed ice skater, who was unable to contract because of heavy weights in her hands.

11. Coriolis's legacy

In 1836 Coriolis was elected into the Academie de Science and in 1838 became deputy director at the École Polytechnique. In 1843 his health deteriorated and he died while working on a revision of his 1829 book. The importance of his work was not realized outside mechanics until 1859 when the French Academy organized a discussion concerning the effect of the earth's rotation on water currents like rivers (Khrgian 1970, 222; Kutzbach 1971, 92; Gill 1982, 210, 371). Coriolis's name began to appear in the meteorological literature at the end of the nineteenth century, although the term "Coriolis force" was not used until the beginning of this century.

All major discoveries about the general circulation and the relation between the pressure and wind fields were made without any knowledge about Gaspard Gustave Coriolis. Nothing in today's meteorology would have been different if he and his work had remained forgotten. The "deflective force" would just have been named after Foucault or Ferrel. (The letter f for the "Coriolis parameter" may be an early attempt to honor one of them.) However, had Coriolis's work been read and his dynamic approach understood, today's confusion about the deflective force and the role of inertia in the atmospheric processes might have
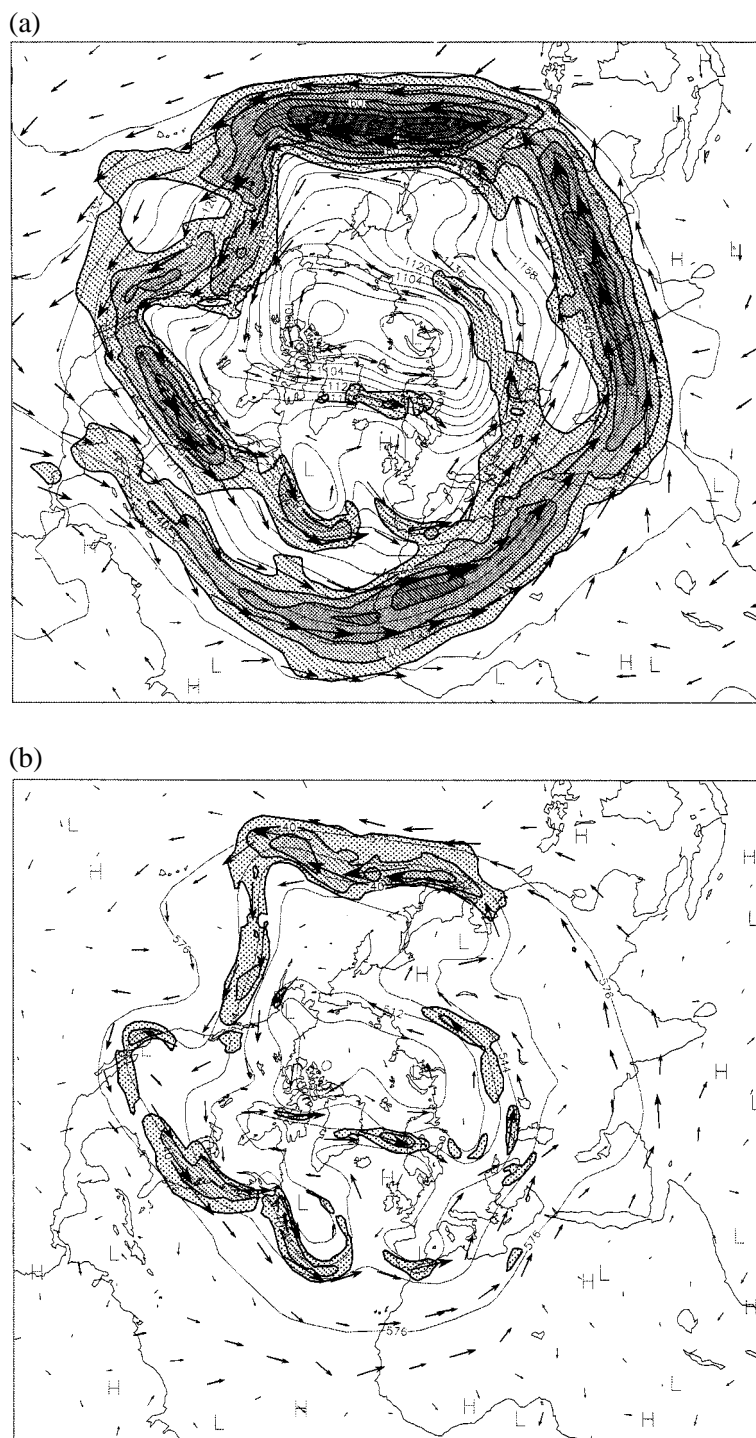
(a)

(b)

FIG. 10. The large-scale circulation at 500 (a) and 200 hPa (b) on 6 January 1997 at 1200 UTC. The jet streams are not only stronger at 200 hPa than at 500 hPa, but there is a strong subtropical jet at 200 hPa over northern Africa and southern Asia, which is not found at 500 hPa. At around this time, two famous European balloonists, R. Branson and P. Lindstrand, took off from Morocco in an attempt to sail around the world by taking advantage of the subtropical jet stream.

been avoided. So he is well qualified to lend his name to the Coriolis force. Had he been with us today, he might have been one of the few who had understood and taught it properly!

Appendix: A nonkinematic look at the jet stream circulation

At the right entrance of the jet the wind is accelerated by the pressure gradient force and is deflected to the right by the Coriolis force; at the exit the pressure gradient force weakens, and the Coriolis force drives the wind to the right, up the pressure gradients and the wind is decelerated by the pressure gradient force. As potential energy converts into kinetic energy at the entrance, the pressure gradient weakens; as kinetic energy converts back to potential energy at the exit, the pressure gradient strengthens—the whole pressure pattern moves downstream.

The downgradient transport of mass at the entrance of the jet stream becomes part of a full three-dimensional circulation where rising motion occurs for warm air, sinking for cold. This means an overall lowering of the point of gravity, which is consistent with the loss of potential energy. The upgradient transport at the jet exit forces for similar reasons a three-dimensional circulation where cold air is rising and warm air sinking, which rises the point of gravity in consistence with the increase in potential energy. The tendency of rising motion at the right entrance and the left exit of the jet streams are favorable locations for synoptic developments.

The flow through the core of the jet is roughly geostrophic and uneffected by any net force, since the pressure gradient force and the Coriolis force balance each other. Depending on the curvature of the trajectory of the air parcels passing through the jet the velocity is supergeostrophic (anticyclonic trajectory) or subgeostrophic (cyclonic trajectory). The generation of supergeostrophic winds results in wind oscillations with a period of about one pendulum day and explains the cycloid shape of most anticyclonic jet streams.

The flow through the jet stream carries rapidly both mass and energy from one end of the jet to the other. Baroclinic, kinetic energy-generating processes at the right entrance, including latent heat releases from a developing cyclone (or hurricane in extreme cases), rapidly affect the conditions at the left exit and can even initiate a development.

With several jet streams lined up after each other, the impact can spread downstream like a "domino effect" from one cyclone to the next. The well-known notion of "downstream development" in the midlatitude westerlies, sometimes described in kinematic or graphical terms of "group velocity," is essentially a matter of energy transfer and transport.

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